MARKETS, INFORMATIONS AND THEIR FRACTAL ANALYSIS

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Market Theories

- Capital Market Theory

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- Speculator bets that the current price of a security is above/below its future value and sells/buys it accordingly at the current price.

Modern Market Theory

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- the covariance of returns is used to explain how diversification is reducing risk

Efficient Market Theory

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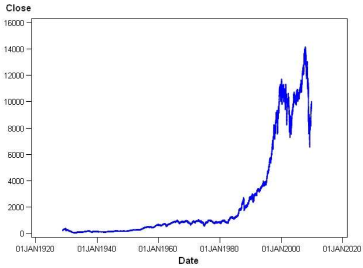
- Classical approach differentiates features of investors trading over many investment horizons.
- The risk for each horizon is the same. Risk and return grow in time.

Statistical characteristics of markets



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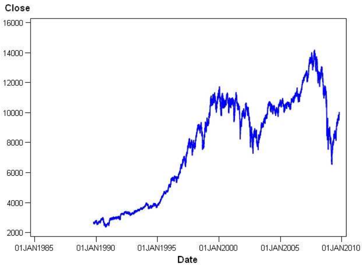

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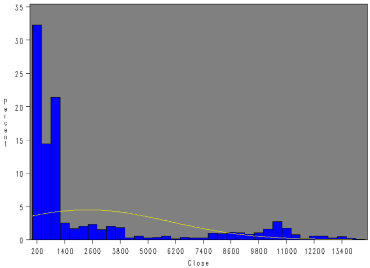

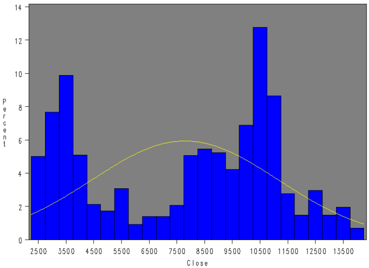

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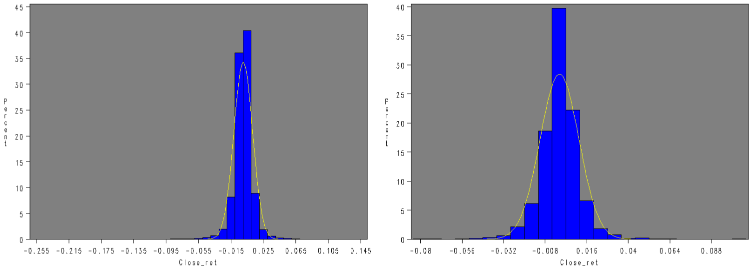

Traders need

- information

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- The market is stable when it consists of investor covering a large number of investment horizons. This ensures that there is ample liquidity for traders.
- The information set is more related to market sentiment and technical factors in the short term than in the longer term. As investment horizons increase, long term fundamental information dominates.

Fractal Market Theory

- If an event occurs that makes the validity of fundamental information questionable, long term investors either stop participating in the market or begin trading based on the short term information set. When the over-all investment horizon of the market shrinks to a uniform level, the market becomes unstable.

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- Prices reflect a combination of short-term technical trading and long-term fundamental valuation.
- If a security has no tie to the economic cycle, then there will be no long-term trend. Trading, liquidity, and short term information will dominate.

What is fractal

Definition

Fractal is attractor (limiting set) of a generating rule (information processor), when the information is generated randomly.

Properties of the fractals

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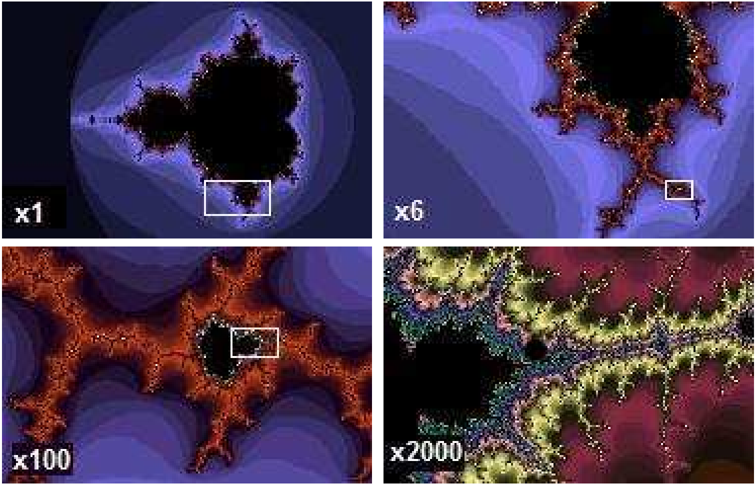
- have the fractional (fractal) dimension.

Self-similarity of the fractals



Types of the fractals

deterministic (they are symetric in generally and they are generated by deterministic rules)

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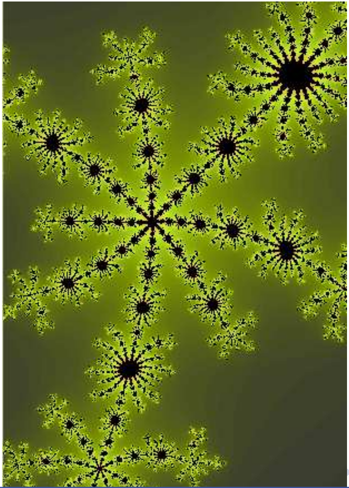
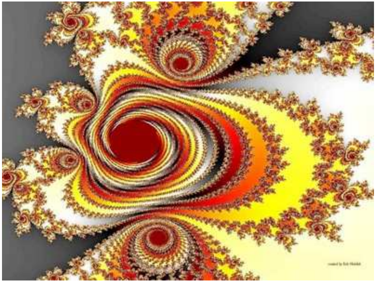
Natural fractals

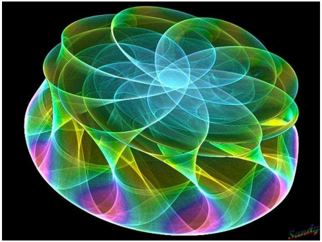
Geometric fractals

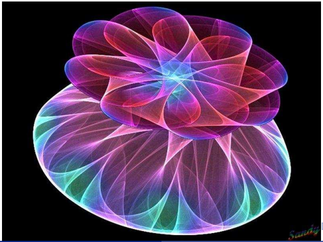


Geometric fractals



Complex fractals

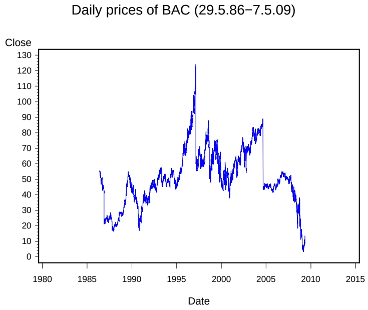


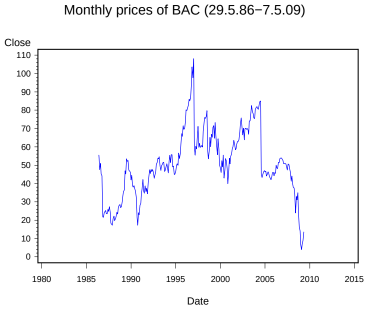


Types of the fractals

random (they do not necessarily have pieces that look like pieces of the whole and they are created by iterating a simple rule to create a self-similar object with a fractal dimension)

Random Fractals





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- are generated by fractional Brownian motion

R/S analysis (Edgard E. Peters 1996)

- $X_{t,N} = \sum_{u=1}^{t}(e_u - M_N)$, where $X_{t,N}$ is cumulative deviation over N period, e_u is influx in period u, M_N is average e_u over N periods.

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- $R/S = (a \cdot N)^{2-D}$, where R/S is rescaled range, N is number of observations, a is constant, D is fractal dimension

Importance of the fractal dimension time series

The fractal dimension recognizes that process can be somewhere between

1. $D = 1.00$ deterministic process
2. $D = 1.50$ random process
3. $1.50 < D < 2.00$ antipersistent time series
4. $1.00 < D < 1.50$ persistent time series
5. $D = 2.00$ proces with normal distribution

Antipersistent (ergodic, mean reverting) time series

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9. the strength of antipersistent behavior depends on how close D is to 2.

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  3. its probability density function can be the normal curve, but it does not have to be.

Persistent or trend-reinforcing time series

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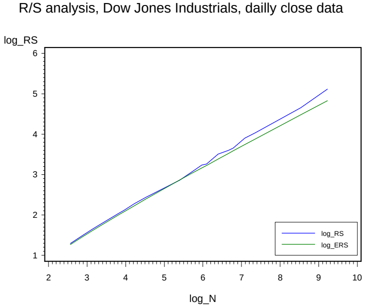
4. persistent time series are fractional brownian motion
5. the strength of the bias depends on how far D is bellow 1.5

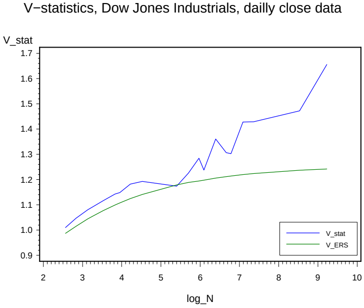
Fractal analysis of the selected financial time series

We applying R/S analysis to the

1. daily close prices of Dow Jones Industrials for consecutive observations during two periods from October 1928 to October 2009 and from October 1989 to October 2009
2. daily log returns of Dow Jones Industrials for consecutive observations during two periods from October 1928 to October 2009 and from October 1989 to October 2009

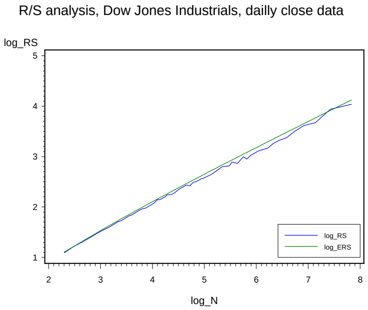
R/S analysis and V-statistics of the daily close prices of Dow Jones Industrials

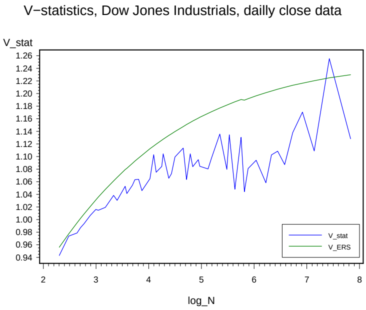




period 24.10.1928-14.10.2009

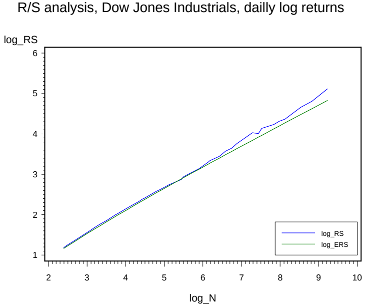
R/S analysis and V-statistics of the daily close prices of Dow Jones Industrials

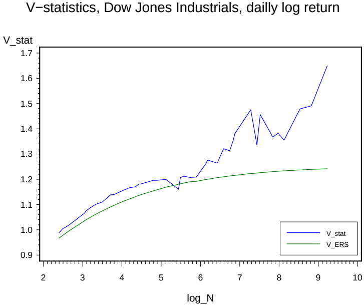




period 17.10.1989-14.10.2009

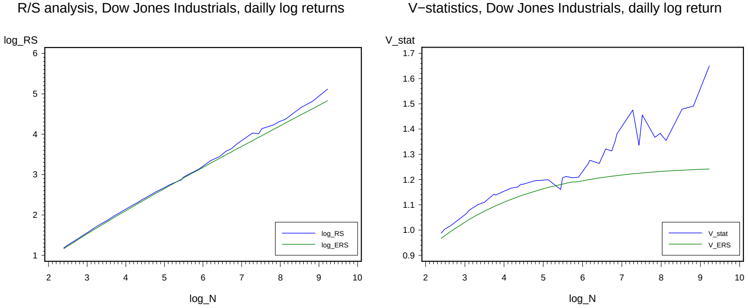
R/S analysis and V-statistics of the daily log return of Dow Jones Industrials





Period: 24.10.1928-14.10.2009

R/S analysis and V-statistics of the daily log returns of Dow Jones Industrials



period 16.10.1989-14.10.2009

Conclusion

Fractal analysis of the finance time series may be usefull for investor.
Our analyzed data have persistent character.
It means for investor, that risk grow with time and they must actively trade with their financial instruments.

Thank You for Your attention.