MARKETS, INFORMATION AND THEIR FRACTAL ANALYSIS

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Abstract: We will summarize the impact of the conflict between randomness and determinism on the markets analysis, in this paper. On the one hand, the market analysts assume that the market is perfectly deterministic, and on the other, there is a group of the analysts that believe the market is completely random. We discuss also information and investment horizons, stability, and risk. Information is processed differently at various frequencies: there will be trends and cycles at all investment horizons. The information can be stochastic, or it can be nonlinear deterministic, but always their exact structure of the trend is time varied. It is predictable, if it keeps the markets stable. Singularities of the market behavior can be explained by chaos theory and fractal statistics. Fractal analysis helps us to understand how markets and economies perform.

Key words: financial time series modeling, fractal, Hurst exponent, fractal Brownian motion.

Introduction

The well–known traditional Capital Market Theory is based on fair games of chance. Under this theory we can model the speculation by probabilities. This theory extends back to Bachelier (1900) and continues to be interesting to this day[1]. This theory gives us the view of the speculator. Speculator bets that the current price of a security is above/below its future value and sells/buys it accordingly at the current price. The speculation involves betting, which makes investing a form of gambling. More recently Markowitz (1952, 1959) used wheels of chance to explain standard deviation. Standard deviation is a measure of risk and the covariance of returns could be used to explain how diversification is reducing risk. Theory of speculators did not differentiate between short term speculators and long term investors. Classical investment theory assumes that the markets are “efficient”. This means, that all prices reflected all current information that could anticipate future events. Therefore, only the speculative, stochastic component could be modelled, the change in prices due to changes in value could not. If market returns are normally distributed “white” noise, then returns are the same at all investment horizons[2]. Classical approach differentiates features of investors trading over many investment horizons. The risk for each horizon is the same. Risk and return grow in time. There is no advantage to begin a long–term investment. In addition, price changes are determined primarily by speculators. Therefore forecasting changes in economic values would not be useful for speculators. This theory assumes usually, that markets follow a random walk. But if markets do not follow a random walk, it is possible

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[1] Edgar E. Peters: *Fractal Market analysis. Applying chaos theory to investment & economics.* John Wiley & Sons, Inc., New York, p.19

[2] Edgar E. Peters: *Fractal Market analysis. Applying chaos theory to investment & economics.* John Wiley & Sons, Inc., New York, p.20

that they may be predicted or we may understand our risk and return potential from investing versus speculating. Fama (1960) introduced Efficient Market Hypothesis (EMH). According to this theory, discontinuities in the price structure and the fat tails are explained so that the market information arrives in discontinuous manner. Investors still react to information homogeneously. Another important assumption is independence. But the people do not make their decisions by this way. For investors some information has obvious implications and then the market can, and often does, make a quick judgment. Other information is not easily valued, particularly if the data are noisy. The noise can be due either to volatility in the particular indicator for structural reasons, or to measurement problems. Both contribute to the inability of the marketplace to uniformly value the information. There is another possibility: new information may contribute to increased levels of uncertainty, rather than increased levels of knowledge or new information increases knowledge of current conditions and facilitates judgment about the future. In traditional theory, information is treated as a generic item and investor is also generic. This generic approach, where information and investors are general cases, also implies that all types of information impact all investor equally. That is, where it fails. The market is made up of many individuals with many different investment horizons. The information has a different impact on different investment horizons. The day trader will be more concerned with technical information. Technical trends are convenient for short–term investor. Traders, who are long–term investors, deal more with the economic cycle. The impact of information is largely dependent on each individual's investment horizons. When the market is composed of many investors with many different investment horizons, it is available liquidity and the markets are stabile. However, when the market loses this structure and all investors have the same investment horizon, then the market becomes unstable, because there is no liquidity. The loss of long–term investors causes the entire market to trade based on the same information set, which is primarily technical. Market horizon becomes short–term when the long–term horizon becomes highly uncertain. Long–term investors either stop participating or they become short–term investors and begin trading on technical information. Market stability relies on diversification of the investment horizons of the participants. The risk should be equal, at each investment horizon. If this is true, then frequency distribution of return is equal. The market follows random walk, which is characterized by the normal distribution. But the shape of frequency distribution is high–peaked and fat–tailed, when compared to the normal distribution. The fat tails occur because a large event occurs through an amplification process. When the large events occur, they tend to be abrupt and discontinuous. The frequency distribution is then discontinuous. In the markets, fat tails are caused by crashes and stampedes, which tend to be abrupt and discontinuous, as predicted by the model.

Another market hypothesis was introduced by Peters (1991) and it is known as Fractal Market Hypothesis (FMH). Fractal Market Hypothesis is capital–market theory that combines fractals and other concept from chaos theory with the traditional quantitative methods to explain and predict market behavior. FMH takes into account the daily randomness of the market and anomalies such as market crashes and stampedes.

Fractal Market Hypothesis proposes the following[3]:

1. The market is stable when it consists of investor covering a large number of investment horizons. This ensures that there is ample liquidity for traders.

[3] Edgar E.Peters: *Fractal Market analysis. Applying chaos theory to investment & economics.* John Wiley & Sons, Inc., New York, p. 49

2. The information set is more related to market sentiment and technical factors in the short term than in the longer term. As investment horizons increase, long term fundamental information dominates.
3. If an event occurs that makes the validity of fundamental information questionable, long term investors either stop participating in the market or begin trading based on the short term information set. When the over–all investment horizon of the market shrinks to a uniform level, the market becomes unstable.
4. Prices reflect a combination of short–term technical trading and long–term fundamental valuation.
5. If a security has no tie to the economic cycle, then there will be no long–term trend. Trading, liquidity, and short term information will dominate.

Fractal Market Hypothesis says that information is valued according to the investment horizon of the investor. Because the different investment horizons value information differently, the diffusion of information will also be uneven. At any one time, prices may not reflect all available information, but only the information important to that investment horizon. Fractal Market Hypothesis gives an economic and mathematical structure to fractal market analysis. Through the Fractal Market Hypothesis we can understand the behavior of the markets.

Motivated by these market hypotheses, our aim is to investigate the performance of Dow Jones Industrials Average in this paper.

Analysis of Dow Jones Industrial Average

Dow Jones Industrial Average (DJIA) [4] is index that is widely used since 1888. We take into account daily and weekly data without holidays from October 1928 to May 2010. Our data files are most complete; they have a large number of observations and covers a long time period. This long period allows to us, learn much about behaviour of the market.

EMH assumes normal distribution of the data. It is known, that actual behaviour of the stock prices does not follow normal distribution. Figure 1 and Figure 2 show daily close prices and daily log returns of Dow Jones Industrial Average for consecutive observations during three periods from 1st October 1928 to 19th May 2010 (contains 20500 data points), from January 1990 to May 2010 and from January 2000 to May 2010. Figure 3 shows the behavior of the distribution of daily log returns of Dow Jones Industrial Average during the same periods. Normal distribution is shown for comparison. Distributions of the log returns are characterized by a high peak at the mean and fatter tails than the normal distribution. These distributions are very similar, but they are not normal. The tails are not only fatter than in normal distribution, but they are uniformly fatter.

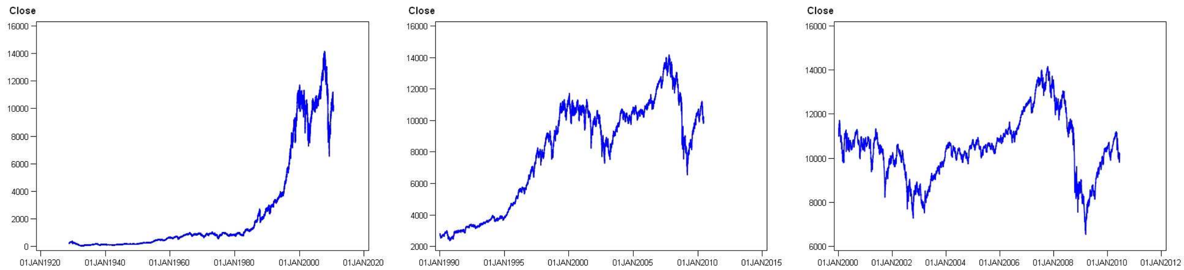

Figure 1: Daily close prices of DJIA (1928–2010, 1990–2010, 2000–2010)

[4] Data follow from www.yahoo.finance.com

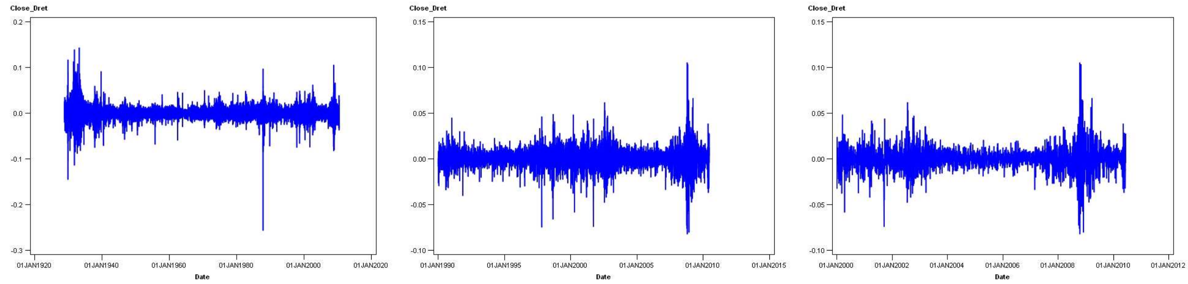

Figure 2: Daily log returns of DJIA (1928–2009, 1990–2010, 2000–2010)

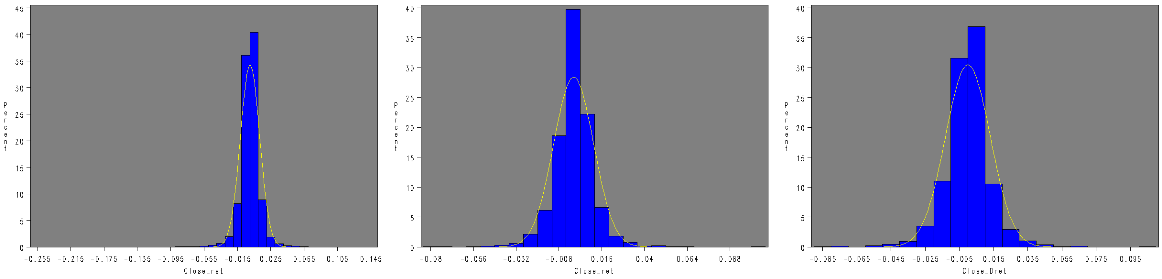

Figure 3: Distribution of daily log returns of DJIA (1928–2010, 1990–2010, 2000–2010)

Further study is based on Peter's work[5]. We will use his methodology for validate EMH. We compute Hurst coeficient H and his expected value $E(H)$ using R/S analysis and we will verify null hypothezis: *The time series is random walk*. If Hurst exponent H and his expected value $E(H)$ is approximately equals, it means, the time series is independent and random during analysed period (Hurst exponent is insignificante). If Hurst exponent H is greater (smaller) then his expected value $E(H)$, time series is persistent (antipersistent) (Hurst exponent is significante). If the series exhibits persistent character, then the time series has long memory and the ratios R/S_n will be increasing. If the ratios R/S_n will be decreasing the time series will be antipersistent. The „breaks" may to signalize a periodic or nonperiodic component in the time series with some finite frequency. We calculated the V–statistics[6] for precisely estimating where this break occurs.

Table 1 and Figure 4 show results of the R/S analysis[7,8] of the Dow Jones Industrial Average log returns during period 1.10.1928–19.5.2010. Hurst coeficient H is equal to 0.5614. Expected Hurst exponent is equal to $E(H)$=0.5325. Standard deviation of $E(H)$ is 0.0070 for 20500 observations. The Hurst exponent for daily log return DJIA is 4.14 standard deviations away from its expected value. This is highly significant result significant at the 95% level. The time series has persistent character. Also plotted is $E(R/S_n)$[9] (green line) as a comparison against the null hypothesis that the system is random walk. There is clearly a systematic deviation from the expected values. However, breaks in R/S graph (see Figure 4) appear to be at 125, 250 and 1025 observations (log125=4.828, log 250=5.521, log 1025=6.932). On the Figure 4 V–statistics clearly stops groving at n=125, n=250 or n=1025 observations. These „breaks" may be signal of a periodic or nonperiodic component in the time series with

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[5] Edgar E. Peters: *Fractal Market analysis. Applying chaos theory to investment & economics.* John Wiley & Sons, Inc., New York, p. 112-131

[6] $V_n = (R/S)_n \big/ \sqrt{n}$, where n is number of observation

[7] Edgar E. Peters: *Fractal Market analysis. Applying chaos theory to investment & economics.* John Wiley & Sons, Inc., New York, p. 53

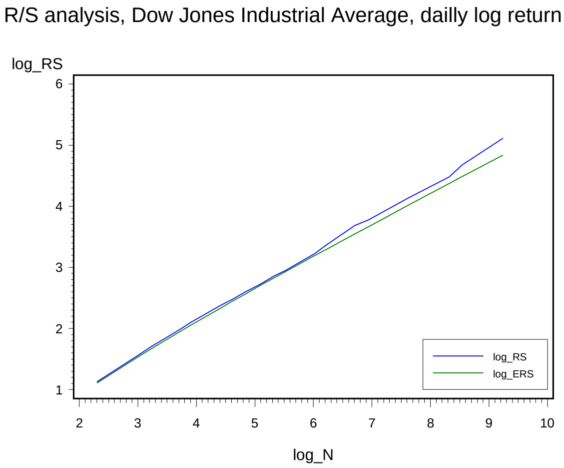
[8] Bohdalová, M. - Greguš, M.: *Financial Markets during the Economic Crisis.* Refereed Program of the E-Leader Conference at Tallin, Estonia, http://www.g-casa.com, ISSN 1935-4819, Chinese American Scholars Association, New York, New York, USA, June 2009.

[9] $E(R/S_n)$ is expected value of the adjusted range

frequency of approximately 125 periods. We will run regression to estimate Hurst exponent for R/S$_n$ values in the next subperiods: n<125, 125≤ n ≤10250, 10<n<250, 250≤ n ≤10250, 10<n<1025 and 1025≤ n ≤10250. Table 2 shows results of the regression analysis for estimating Hurst exponents and their expected values during analyzed periods. During periods for n<125 and 10<n<250 the time series has random character. Hurst exponent is insignificant. H is for n<125 only 1.075 standard deviations away from its expected value and 0.184 standard deviations away from its expected value for 10<n<250. During periods for 125≤ n ≤10250 and 1025≤ n ≤10250 the time series has persistent character. Hurst exponent is significant. It means, ancient history had random character and recent history has long memory effect.

n	*Log N*	*Log RS*	*RS*	*E*(*RS*)	*V stat*	*V E*(*RS*)
10	2.303	1.129	3.093	3.023	0.978	0.956
20	2.996	1.555	4.737	4.611	1.059	1.031
25	3.219	1.695	5.449	5.258	1.090	1.052
41	3.714	1.981	7.247	6.987	1.132	1.091
50	3.912	2.101	8.174	7.813	1.156	1.105
82	4.407	2.374	10.742	10.277	1.186	1.135
100	4.605	2.472	11.846	11.453	1.185	1.145
125	4.828	2.592	13.360	12.924	1.195	1.156
164	5.100	2.729	15.324	14.953	1.197	1.168
205	5.323	2.854	17.362	16.841	1.213	1.176
250	5.521	2.950	19.108	18.707	1.208	1.183
410	6.016	3.222	25.074	24.212	1.238	1.196
500	6.215	3.361	28.819	26.860	1.289	1.201
820	6.709	3.684	39.805	34.724	1.390	1.213
1025	6.932	3.770	43.394	38.960	1.355	1.217
2050	7.626	4.139	62.714	55.581	1.385	1.228
4100	8.319	4.481	88.359	79.086	1.380	1.235
5125	8.542	4.673	107.012	88.558	1.495	1.237
10250	9.235	5.112	166.012	125.723	1.640	1.242

Table 1: R/S analysis of the daily log return of DJIA, 1928–2010



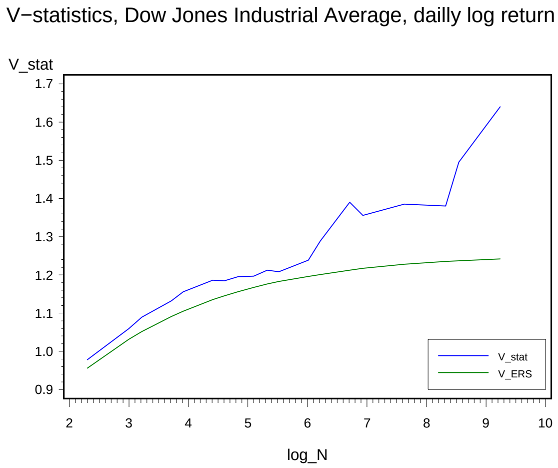


Figure 4: R/S analysis and V statistics of the daily log return of DJIA, (1928–2010)
H=0.5614, E(H)= 0.5325, significance =4.14

	DJIA	E(R/S)	DJIA	E(R/S)	DJIA	E(R/S)	DJIA	E(R/S)	DJIA	E(R/S)
	10<n<125		125≤ n ≤10250		10<n<250		250≤ n≤10250		1025≤ n≤10250	
Intercept	–0.198	–0.206	–0.153	0.079	–0.142	–0.170	–0.181	0.103	–0.265	0.137
Hurst	**0.584**	**0.577**	**0.565**	**0.516**	**0.567**	**0.565**	**0.569**	**0.513**	**0.578**	**0.509**

Exponent										
Standard Error	0.015	0.011	0.032	0.006	0.021	0.014	0.037	0.004	0.045	0.002
R squared	0.999	1.000	0.999	1.000	0.999	0.999	0.998	1.000	0.994	1.000
Number of observation	7	7	12	12	10	10	9	9	5	5
Significance	1.075		7.023		0.184		7.962		9.905	

Table 2: Regression results, DJIA, estimation of the Hurst exponent, (1928–2010, daily data)

We have found that the daily log return of the DJIA has one periodic cycle with length 125 days or 25 weeks and one nonperiodic cycle with length 1025 days or about four years. Next, we have analyzed recent history – during period 1990–2010 and period 2000–2010. These periods have random character. Period 2000-2010 is very tumultuous. It includes depression. The periods are influenced by political events, price controls, etc. on the market.

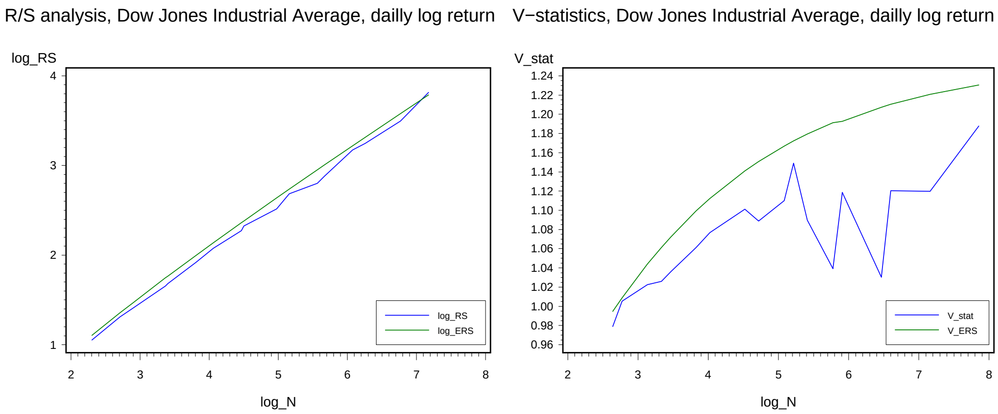


Figure 5: R/S analysis and V statistics of the daily log return of DJIA, 1990–2010, H=0.54093, $E(H)$= 0.550959, significance =-1.16873

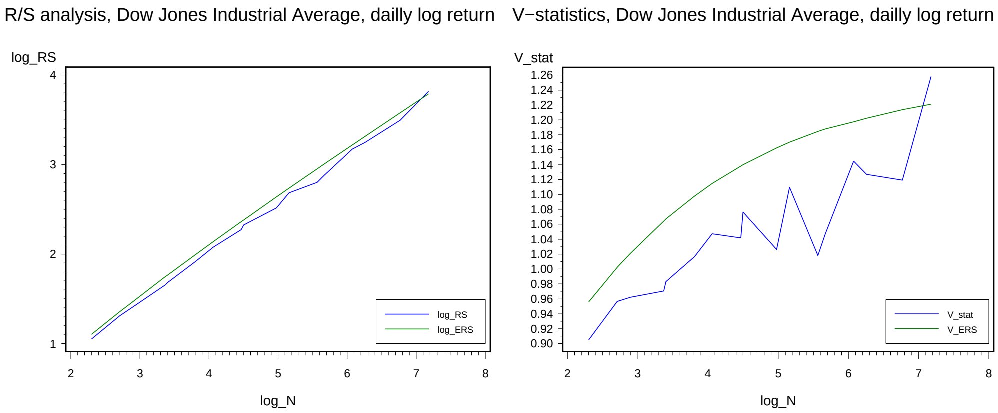


Figure 6: R/S analysis and V statistics of the daily log return of DJIA, 2000–2010, H=0.54748, $E(H)$= 0.550959, significance =-0.06391

Now, we take into account weekly (5 day) log returns of the DJIA and we will verify stability of the Hurst exponent. We see the results of the R/S analysis on the Figure 7, Figure 8 and Figure 9. Analyzed periods are the same as above and the results are similar. Hurst exponent is significant only for period 1928-2010 and insignificant for the rest periods.

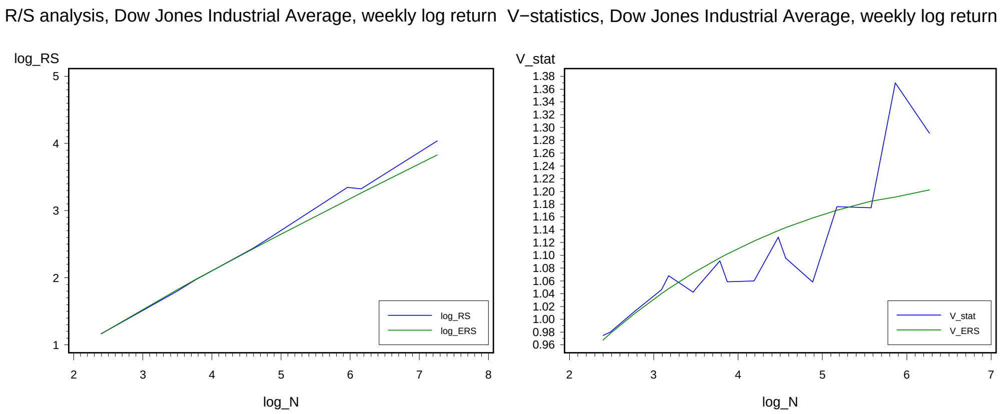


Figure 7: R/S analysis and V statistics of the weekly log return of DJIA, 1928–2010, H= 0.592523, $E(H)$= 0.545382, significance = 3.076863

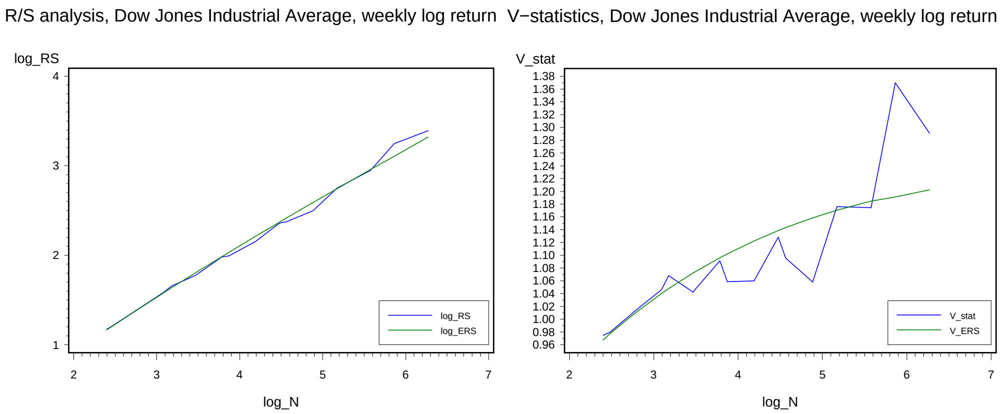


Figure 8: R/S analysis and V statistics of the weekly log return of DJIA, 1990–2010, H= 0.5691, $E(H)$= 0.5559, significance = 0.429719

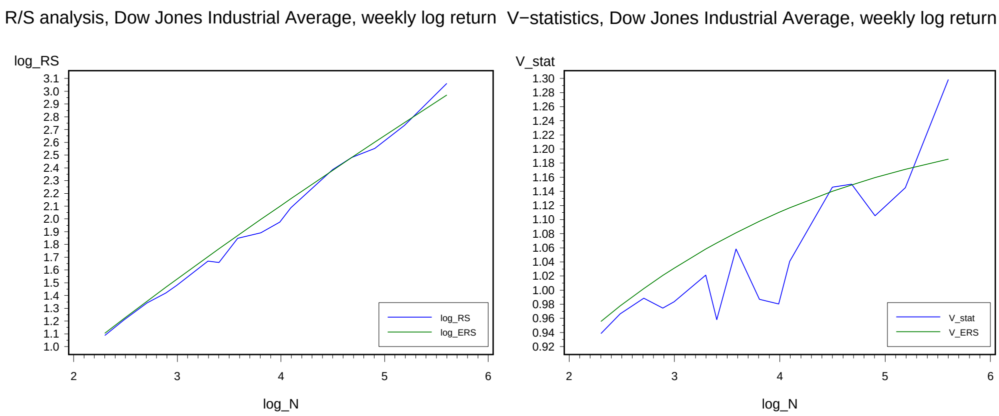


Figure 9: R/S analysis and V statistics of the weekly log return of DJIA, 2000–2010, H= 0.5803, $E(H)$= 0.5649, significance = 0.356434

Conclusion

Information obtained by fractal analysis can be used as the basis of momentum analysis and other forms of technical analysis. The second use is in choosing periods for model development, particularly for back testing.

We have analyzed a very long period. How stable are our findings? Market reacts to information and the way it reacts is not very different from the way it reacted in the 1930s, even though the type of information is different. Therefore the underlying dynamics and, in particular, the statistics of the market have not changed. This would be especially true of fractal statistics.

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